REPLACEMENT MODELS

- The replacement policy consists of calculating the increased operating cost, forced idle time cost together with the cost of replacing with new equipment.
- Also, replacement of items such as electric bulbs,radio tubes etc which does not deteriorate with time but fail suddenly.

MODELS:

- Replacement of items that deteriorate i.e.whose maintenance costs increase with time
- Replacement of items that deteriorate i.e.whose maintenance costs increase with time and value of money also changes
- Replacement of items that fail suddenly

-- Individual repl policy in which an item is replaced immdly after it fails

-- Gp repl policy in which all items are replaced whether they have failed or not ith a proviso that if any item fails before the optimal time,it may be individually replaced

REPLACEMENT MODELS

(a) When t is a continuous variable

Let, C= Capital Cost of Item

S= Scrap Value

Tavg= Avg. annual cost of item

n= no. of yrs item is to be in use

f (t) = operating & maint cost of item at time t

To find n that minimises T (n) = Total cost incurred during n years

Annual cost of item at any time t = $C-S+\int_0^n f(t)\,dt$

Avg annual cost = Tavg = $\frac{1}{n}\left\{ (c-s) + \int_0^n f(t)\,dt \right\}$ --- (1)

- diff wrt n (for Tavg to be min) and equating to zero
- $$\frac{d}{dn}(T_{avg}) = \frac{d}{dn}\left[\frac{1}{n}(c-s)\right] + \frac{d}{dn}\left[\frac{1}{n}\int_0^t f(t)\,dt\right]$$
- $$= \frac{-1}{n^2}(c-s) + \left[\frac{f(n)}{n} - \frac{1}{n^2}\int_0^n f(t)\,dt\right] = 0$$
- Or $$\frac{1}{n^2}(c-s) + \frac{1}{n^2}\int f(t)\,dt = \frac{f(n)}{n}$$

$$f(n) = \frac{1}{n}\left[(c-s) + \int_0^n f(t)\,dt\right] = T_{avg} \text{ from (1)}$$

- $f(n) = \frac{1}{n} [(c-s) + \int f(t)\, dt]$ = Tavg from (1)

- Items should be replaced when avg annual cost becomes equal to current maint cost.

- (b) When t is a discrete variable

- $$T(n) = (C-S) + \sum_{0}^{n} f(t)\, dt$$

- (Total cost incurred during nyrs)

- Avg annual cost incurred on item= $\frac{1}{n}$ [(C-S $\sum f(t)dt$]

Without proof we can state that n is optimal at least avg annual cost

PURCHASE PRICE = Rs. 7000 = C

YEAR	1	2	3	4	5	6	7	8
MAINT COST	900	1200	1600	2100	2800	3700	4700	5900
RESALE VALUE	4000	2000	1200	600	500	400	400	400

WHEN SHOULD MACHINE BE REPLACED

YEAR OF SERVICE	RESALE VALUE	C-S	ANNUAL MAINT COST f (t)	$\sum f(t)$	T.C. $[C-S)+\sum^{n} f(t)]$	AVG ANNUAL COST $\frac{1}{n}[C-S+\sum^{n} f(t)]$
1	4000	3000	900	900	3900	3900
2	2000	5000	1200	2100	7100	3550
3	1200	5800	1600	3700	9500	3166.67
4	600	6400	2100	5800	12200	3050
5	500	6500	2800	8600	15100	3020
6	400	6600	3700	12300	18900	3150
7	400	6600	4700	17000	23600	3371.43
8	400	6600	5900	22900	29500	3687.50

MACHINE TO BE REPLACED AT END OF 5 YRS

TIME VALUE OF MONEY IS CONSIDERED

$F = P\,(1+i)^n$ or $P = \dfrac{F}{(1+i)^n} = F\,(p/f, v\,\%, n)$

(1+t)

$V = \dfrac{1}{(1+i)}$ or $V^r = \dfrac{1}{(1+i)^r}$

This is called discounting factor

0 1 2 3 n-1 n

R1 R2 R3 Rn

C= PURCHASE PRICE OF M/c . R1, R2 ---- Rn = RUNNING COST IN 1st, 2nd ----- n th year of machine.

PAYMENTS ARE MADE AT BEGINNING OF EACH YEAR

$$P.W. = C + R_1 + \frac{R_2}{(1+i)^1} + \frac{R_3}{(1+i)^2} + \text{---------} \frac{R_n}{(1+i)^{n-1}}$$

$$= C + R_1 + R_2V + R_3V^2 + \text{---}.\ R_n V^{n-1}$$

WE CONCLUDE THAT

$$R_{n+1} > \frac{C+R_1+R_2V + R_3V^2+ - - - - R_n. V^{n-1}}{1+V+V^2+ - - - - - - - - - - V^{n-1}}$$

$$R_{n+1} > \frac{C+ \sum_{r=1}^{n} R_r V^{r-1}}{\sum_{r=1} V^{r-1}}$$

M/c SHOULD BE REPLACED IF NEXT PERIOD COST IS GREATER THAN THE WEIGHTED AVG OF PREVIOUS COSTS

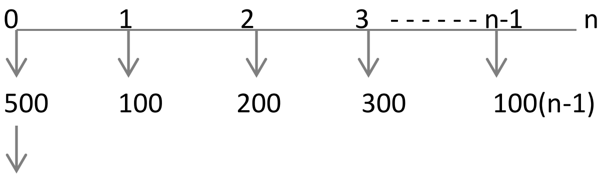
$$\text{IF} \quad R_{n+1} < \frac{C+ \sum_{r=1}^{n} R_r V^{r-1}}{\sum_{r=1} V^{r-1}}$$

M/c SHOULD NOT BE REPLACED

COST OF MACHINE = Rs. 500

OP & MAINT COST = 0 IN FIRST YEAR & INCREASES BY Rs. 100 EVERY YEAR

R= 5%, WHEN SHOULD THE M/c BE REPLACED



DISCOUNT RATE = $\frac{1}{1+r} = \frac{1}{1+0.05} = 0.9524$

YR OF SERVICE	MAINT COST Rr	DISCOUNT FACTOR V^{r-1}	DISCOUNT COST R_rV^{r-1}	CUM. TOTAL DISCOUNTED COST $C+\sum^{n} R_rV^{r-1}$	DIVIDING FACTOR $\sum^{n} V^{r-1}$	WEIGHTED AVG ANN. COST $\frac{C+\sum^{n} R_rV^{r-1}}{\sum_{r=1} V^{r-1}}$
1	0	1	0	500	1	500
2	100	0.9524	95.24	595.24	1.9524	304.88
3	200	0.9070	181.40	776.64	2.85	217.61
4	300	0.8638	259.14	1035.78	3.72	278.20
5	400	0.8227	329.08	1364.86	4.54	300.28

MACHINE SHOULD BE REPLACED AFTER 3rd YR AS 300 > 217.61

PROBABILISTIC MODELS: ITEMS FAILING COMPLETELY

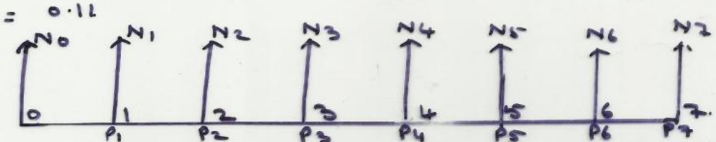
- INDIVIDUAL REPLACEMENT POLICY
- GROUP REPLACEMENT POLICY

FAILURE RATE

END OF WEEK:	1	2	3	4	5	6	7
PROB. OF FAILURE:	0.07	0.18	0.30	0.48	0.69	0.89	1.00

$P_1 = 0.07$, $P_2 = 0.11$, $P_3 = 0.12$, $P_4 = 0.18$, $P_5 = 0.21$, $P_6 = 0.20$

$P_7 = 0.11$



LET, N_0 – NO. OF UNITS REPLACED AT END OF WEEK 0 OR BEGINNING OF FIRST WEEK = 100

$N_1 = N_0 \times P_1 = 100 \times .07 = 7$

$N_2 = N_0 \times P_2 + N_1 \times P_1 = 100 \times 0.11 + 7 \times .07 = 12$

$N_3 = N_0 \times P_3 + N_1 \times P_2 + N_2 \times P_1 = 100 \times .12 + 7 \times .11 + 12 \times .7$
$= 14$

$N_4 = N_0 \times P_4 + N_1 \times P_3 + N_2 \times P_2 + N_3 \times P_1$

$= 100 \times .18 + 7 \times .12 + 12 \times .11 + 14 \times .07 = 21$

$N_5 = 27$, $N_6 = 30$, $N_7 = 25$

INDIVIDUAL REPLACEMENT COST

EXPECTED LIFE OF EACH UNIT $= \sum_{i=1}^{7} i \times P_i$

$= 1 \times .07 + 2 \times 0.11 + \cdots 7 \times .11 = 4.39$ WEEKS

AVG NO. OF FAILURES / WEEK $= 100 / 4.39 \approx 23$

$\therefore$ COST OF INDIVIDUAL REPLACEMENT $= 23 \times 9 =$ Rs 207

GROUP REPL. POLICY

END OF WEEK A	COST OF REPL. 100 UNITS AT A TIME B	COST OF REPL UNITS INDIV. DURING REPL. PERIOD C	TOTAL COST B+C = D	AVG COST/WEEK D/A
1	300	7×9=63	363	363
2	300	(7+12)×9=171	471	235.50
3	300	(7+12+14)×9= 297	597	199.00
4	300	(7+12+14+21)×9= 486	786	196.50
5	300	(7+ ---- + 27)×9= 729	1029	205.80
6	300	(7+ ----+30)×9 = 999	1299	216.50
7	300	(7+ ----+ 25)×9= 1226	1526	217.71

{ INDIV REPL COST - Rs 9
SIMULT. REPL COST - Rs 3 }

GROUP REPL PERIOD - 4 WEEKS

INDIVIDUAL REPLACEMENT COST/WEEK = Rs 207.

MIN^m GP REPL COST/WEEK = Rs 196.50 ✓

REPLACEMENT ONCE IN 4 WEEKS & INDIV. REPL OF FAILURES DURING THIS 4 WEEK PERIOD

When does indiv replacement become more economical.

Let x be the gp replacement price for the bulb

Then, $\text{Rs } 207 < \frac{100*x+9(7+12+14+21)}{4}$

Or x> 3.42

The replacement cost per bulb in gp repl policy should be greater than Rs 3.42.In such case indiv repl policy is more economical

#The following mortality rates have been observed in an installation of 1000 bulbs:

End of week:	1	2	3	4	5	6
Prob of failure:	.09	.25	.49	.85	.97	1.00
Indiv prob	.09	.16	.24	.36	.12	.03

Find the cost of

- Individual replacement
- Group replacement
- At what gp replacement price per bulb would individual replacement become preferable

Let Ni = no.of replacements made at end of ith week.

No = 1000

N1 = Noxpi = 1000x.09 = 90

N2 = Noxp2 +N1p1 = 1000x.16+90x.09= 168

N3 =Noxp3+N1XP2+N2P1=1000X.24+90X.16 +168X.09 = 269

N4=Noxp4+N1Xp3+N2Xp2+N3Xp1

= 1000x.36+90x.24+168x.16+269x.09 = 432

N5=Noxp5+N1xp4+N2Xp3+N3Xp2+N4Xp1 =

=274

N6=Noxp6+N1xp5+N2xp4+N3Xp3+N4Xp2+N5xp1

= 1000x.03+90x.12+168x.36+269x.24+432x.16+

274x.09= 260

N7= Noxp7+N1xp6+N2xp5+N3xp4+N4Xp3+N5xp2

+N6Xp1=1000x0+90x.03+168x.12+269x.36+432x

.24+274x.16+260x.09 = 291

No. of bulbs failing increase till the 4^{th} week then decreases and increases again from 7^{th} week onwards.Thus N will continue to oscillate till a steady state is reached

Optimal gp replacement interval

End of week	Total cost of gp repl	Avg cost/week
1	1000x.70+90x3=970	970
2	1000x.70+(90+168)x3=1474	737
3	1000x.7+(90+168+269)x3=2281	760.33
4	1000x.7+(90+168+269+432)x3=3577	894.25

As the avg minimum cost is in 2nd week,it is optimal to have gp replacement after every two weeks

Individual Replacement policy:

Avg (expected)life of light bulbs = $\sum ip_i$

=1x.09+2x.16+3x.24+4x.36+5x.12+6x.03=3.35wks

Avg no. of failures per week = 1000/3.35=299

Cost of indiv repl of bulbs per week = 299x3=Rs897

Cost of gp repl per week = Rs 737

It is advisable to adopt the policy of group replacement

At what cost of gp repl policy will indiv repl become economical

Let y be the gp repl cost per bulb

$$\text{Rs } 897 < \frac{1000*y + 3(90+168)}{2}$$

or y> Rs 1.02